ENEE 623: Digital Communication, Problem Set 1

1. Consider the scalar quantization of a $\mathcal{R}$-valued rv. X with $E[|X|] < \infty$, $E[|X|^2] < \infty$. Let a quantizer $Q : \mathcal{R} \to \mathcal{C} = \{y_1, ..., y_N\} \subset \mathcal{R}$ be such that its codebook $\mathcal{C}$ satisfies the"centroid" condition

Show that

a) $E[X - Q(X)] = 0$, i.e., the quantizer output is unbiased;

b) $E[Q(X)(X - Q(X))] = 0$, i.e., the quantizer output is uncorrelated with the quantization error; and

c) $E[(X - Q(X))^2] = \sigma_X^2 - \sigma_{Q(X)}^2$, i.e., the variance of the quantization error equals the difference of the variances of the signal X and its quantized version.

2. Proakis, Problem 4.19: $\pi/4$-QPSK may be considered as two QPSK systems offset by $\pi/4$ radians.

a) Sketch the signal space diagram for a $\pi/4$-QPSK signal.

b) Using Gray encoding, label the signal points with the corresponding data bits.

3. Proakis, Problem 4.21 (a), (c): A PAM partial response signal (PRS) is generated as shown in Figure P4.21 by exciting an ideal low-pass filter of bandwidth W by the sequence

$$B_n = I_n + I_{n+1}$$

at a rate $\frac{1}{T} = 2W$ symbols/s. The sequence $\{I_n\}$ consists of binary digits selected independently from the alphabet $\{+1, -1\}$ with equal probability. Hence, the filtered signal has the form

$$v(t) = \sum_{n=-\infty}^{\infty} B_n g(t - nT), \quad T = \frac{1}{2W}$$

a) Sketch the signal space diagram for $v(t)$ and determine the probability of occurence of each symbol.

b) Determine the autocorrelation and power density spectrum of the three-level sequence $\{B_n\}$.

c) The signal points of the sequence $\{B_n\}$ form a Markov chain. Sketch this Markov chain and indicate the transition probabilities among the states.

4. Proakis, Problem 4.22: The low-pass equivalent representation of a PAM signal is

$$u(t) = \sum_n I_n g(t - nT)$$

Suppose $g(t)$ is s rectangular pulse and

$$I_n = a_n - a_{n-2}$$

where $\{a_n\}$ is a sequence of uncorrelated binary-valued (1,-1) random variables that occur with equal probability.

a) Determine the autocorrelation function of the sequence $\{I_n\}$.

b) Determine the power density spectrum of $u(t)$.

c) Repeat (b) if the possible values of the a_n are (0,1).

5. Proakis, Problem 4.28(just the phase tree): Sketch the phase tree, the state trellis, and the state diagram for partial response CPM with $h = \frac{1}{2}$ and

$$g(t) = \begin{cases} \frac{1}{4T} & 0 \le t \le 2T \\ 0 & \text{otherwise} \end{cases}$$

6. Proakis, Problem 4.30: Show that 16 QAM can be represented as a superposition of two four-phase constant envelope signals where each component is amplified separately before summing, i.e.,

$$s(t) = G(A_n cos 2\pi f_c t + B_n sin 2\pi f_c t) + (C_n cos 2\pi f_c t + D_n sin 2\pi f_c t)$$

where $\{A_n\}, \{B_n\}, \{C_n\}$, and $\{D_n\}$ are statistically independent binary sequences with elements from the set {+1, -1} and G is the amplifier gain. Thus, show that the resulting signal is equivalent to

$$s(t) = I_n cos 2\pi f_c t + Q_n sin 2\pi f_c t$$

and determine I_n and Q_n in terms of A_n, B_n, C_n and D_n.

7 Benedetto-Biglieri, Problem 6.5: Derive the squared Euclidean distance d_B^2 for partial-response CPM with rectangular pulses and $L = 2$. Compare the values obtained by considering the merges at $t = 3T$ and those at $t = 4T$.

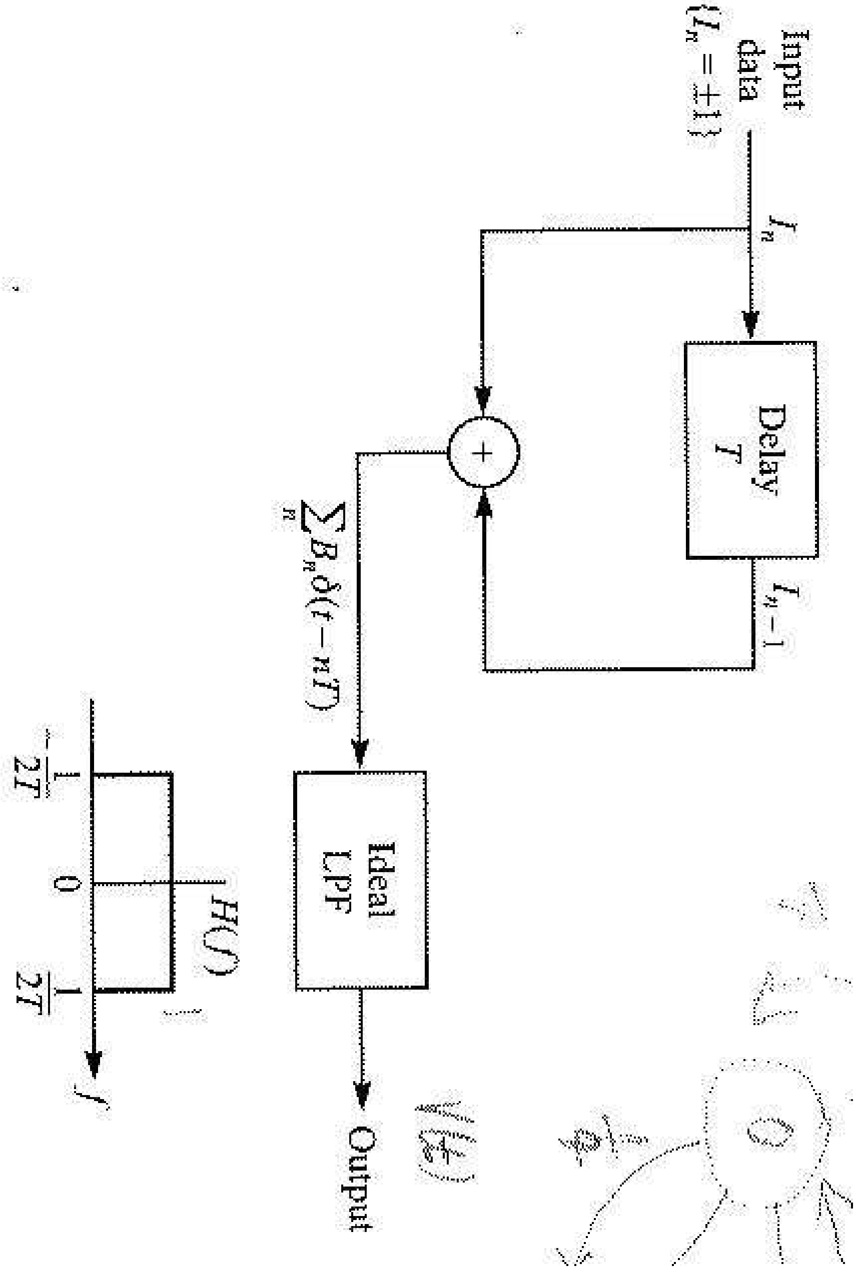

Figure 1: P4.21