Two dimension reciprocal lattice

First, in the equations below, “•” is the vector inner product and “×” is the vector outer product.

A 3D real space lattice is defined by lattice vectors A₁, A₂, and A₃; the vector length is a₁, a₂, and a₃, respectively. The unit vectors are v₁, v₂, and v₃. (Here, we use capital letters to represent a vector, and the small case letters to show the length.) That is,
A₁=a₁v₁;
A₂=a₂v₂; and
A₃=a₃v₃.

The corresponding reciprocal lattice vectors, B₁, B₂, and B₃, are defined as:
B₁=2π(A₂×A₃)/[A₁•(A₂×A₃)]=2π(A₂×A₃)/[V_{cell}]=2π(1/a₁)(v₂×v₃).
So, B₁ is in the direction (v₂×v₃), normal to both A₂ and A₃. The length of B₁ is 2π/a₁.

For a cubic lattice,
B₁=(2π/a₁)v₁;
B₂=(2π/a₂)v₂; and
B₃=(2π/a₃)v₃.

Now, for a 2D lattice defined by A₁ and A₂ only, we can imagine that the z-directional lattice vector is approaching zero in length, i.e., a₃ → 0. We assume that the cell area enclosed by the two lattice vectors A₁ and A₂ is “area.”

By sticking to the same definition as in 3D, we learn that:
B₁=2π(A₂×A₃)/[A₁•(A₂×A₃)]=2π(A₂×a₃v₃)/[A₁•(A₂×a₃v₃)]=2π(a₃)(A₂×v₃)/V_{cell}
=2π(a₃)(A₂×v₃)/[a₃(area)] =2π(A₂×v₃)/area→2πA₂/area, but the vector direction of B₁ is v₂×v₃.

When the same rules are now applied to a 2D square lattice, say, v₁//x, v₂//y, and v₃//z, we have:
B₁=(2π/a₁)v₁;
B₂=(2π/a₂)v₂.

For a 2D rectangular lattice, the above relation holds.

In addition, in 2D, B₁ is always perpendicular to A₂, since we derive the relations in 2D from the convention in 3D, and in 3D B₁ is always perpendicular to A₂ and A₃. That is, in 2D,
B₁•A₂=0; and B₂•A₁=0.

Furthermore, in 2D,
B₁•A₁=2π; and B₂•A₂=2π.