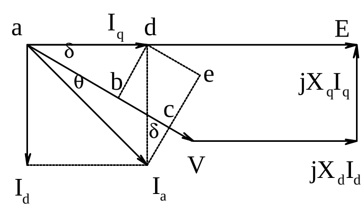
Chapter 3. Generator and Transformer Models; The Per-Unit System (Part II)

3.4 Salient-Pole Synchronous Generators

Due to the non uniform airgap of the salient poles, the flux is stronger in the direct axis than in the quadrature axis. Thus the reactance X_d in the direct axis direction is larger than X_q in the quadrature axis direction. The common way of dealing with this is to decompose the armature current into two components: one along the direct axis and one along the quadrature axis. Now the voltage can be found by adding the components along these directions. It is noted that an Ampere in the direct axis direction will produce more voltage than an Ampere in the quadrature direction (due to the difference in magnitude between X_d and X_q). This is illustrated in the figure below.

Note how the contribution to voltage in the direct axis is much larger than in the quadrature axis. Note also that the arma ture resistance is neglected. It is no longer possible to represent the machine by a simple equivalent circuit.

The excitation voltage is given by:



$$|E| = |V|\cos\delta + X_d I_d$$

The three phase power at the generator is given by:

$$P = 3|V||I_a|\cos\theta$$

Using the figure above we see that

$$\begin{aligned} |I_a|\cos\theta &= ab + de \\ &= I_q\cos\delta + I_d\sin\delta \end{aligned}$$

Therefore the power is: $P = 3|V|\left(I_q\cos\delta + I_d\sin\delta\right)$. From the diagram we see that $|V|\sin\delta = X_q I_q$ or $I_q = \dfrac{|V|\sin\delta}{X_q}$. Also from an equation above we have:

$I_d = \dfrac{|E| - |V|\cos\delta}{X_d}$, therefore the power becomes

$P = 3|V|\left(\dfrac{|V|\sin\delta}{X_q}\cos\delta + \dfrac{|E| - |V|\cos\delta}{X_d}\sin\delta\right)$. Note that $\sin\delta\cos\delta = \dfrac{\sin 2\delta}{2}$, thus:

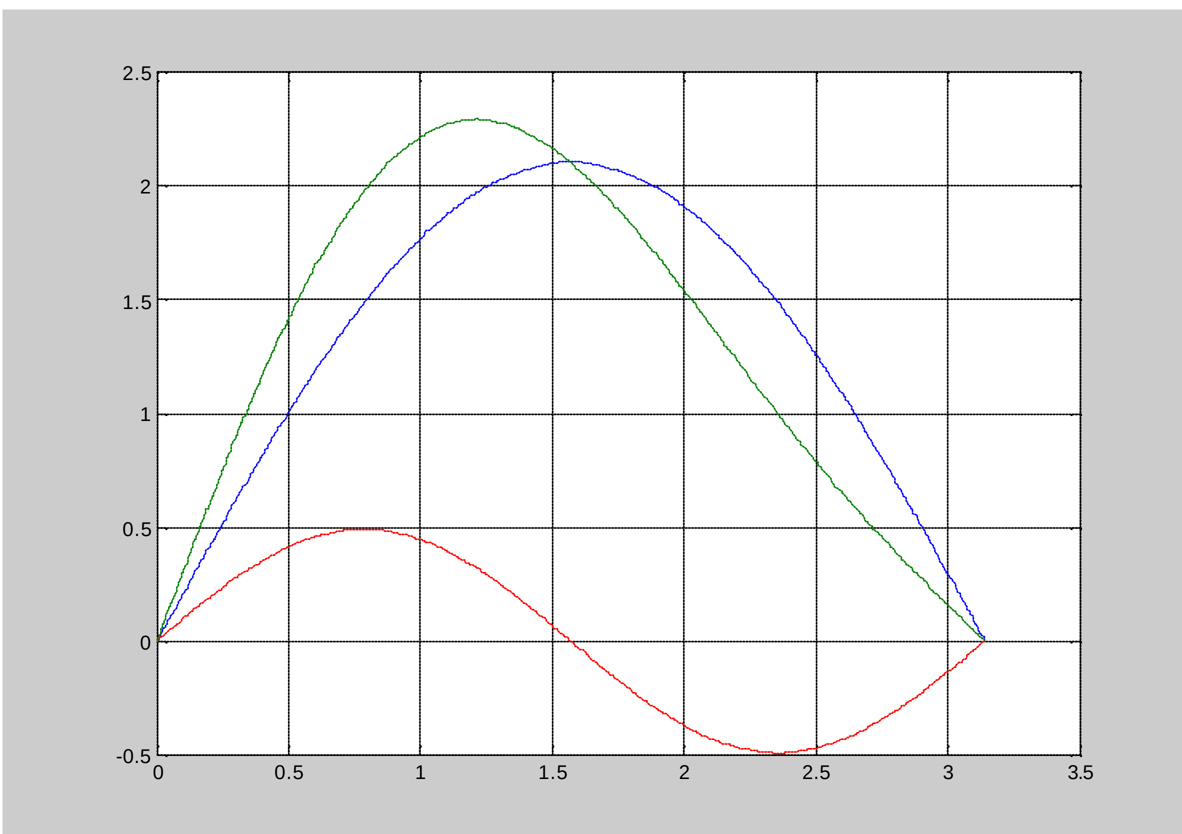
$$P = 3\frac{|E||V|\sin d}{X_d} + \left[\frac{3|V|^2}{X_q} - \frac{3|V|^2}{X_d}\right]\frac{\sin 2d}{2}$$

Which simplifies to the equation in the book (3.31) page 64:

$$P = 3\frac{|E||V|\sin d}{X_d} + 3|V|^2\frac{X_d - X_q}{2X_d X_q}\sin 2d$$

Remarks: the author does not show the details shown above, but the results are well known. Also note that if the machine is cylindrical, then $X_d = X_s$ and $X_q = X_d = X_s$, thus the power equation becomes the normal power, namely: $P = 3\frac{|E||V|\sin d}{X_s}$. Also note that the new term known as the reluctance power is at frequency (in space) twice that of the normal power. Thus the power-angle relationship is changed from a perfect sinusoid. This is shown below:

```
delta=0:.001:pi;
Xs=1; Xd=1; Xq=0.6; E=1; V=0.7;
P=3*E*V*sin(delta)/Xs;
P2=3*V^2*(Xd-Xq)*sin(2*delta)/(2*Xd*Xq);
Psal=P+3*V^2*(Xd-Xq)*sin(2*delta)/(2*Xd*Xq);
plot(delta,P,delta,Psal,delta,P2),grid
```



Note that the salient-pole machine has greater peak power (stiffer), and this peak power occurs at an angle slightly less than $p/2$ radians.

3.6 Equivalent Circuit of a Transformer

First consider an ideal transformer, one with no leakage and infinite permeability. Let the transformer have N_1 turns on the left side and N_2 turns on the right side. Assume the current IN on the left is I_2' and OUT on the right is I_2. In such a case we have (assuming sinusoidal flux $f = \Phi \cos wt$):

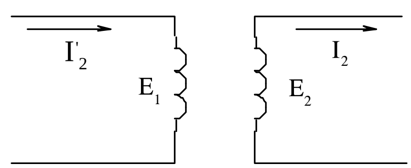
$$
\begin{aligned}
e_1 &= N_1 \frac{df}{dt} \\
&= -wN_1\Phi \sin wt \\
&= E_{1\max} \cos\left(wt + 90^\circ\right)
\end{aligned}
$$

where $E_{1\max} = 2p\, fN_1\Phi$. Thus the RMS value of E_1 is $E_1 = 4.44\, fN_1\Phi$. Since the same flux links both windings, we have on the right: $E_2 = 4.44\, fN_2\Phi$. Also, since the core has no losses, we must have $I_2'N_1 = I_2N_2$. This leads to the relationships:

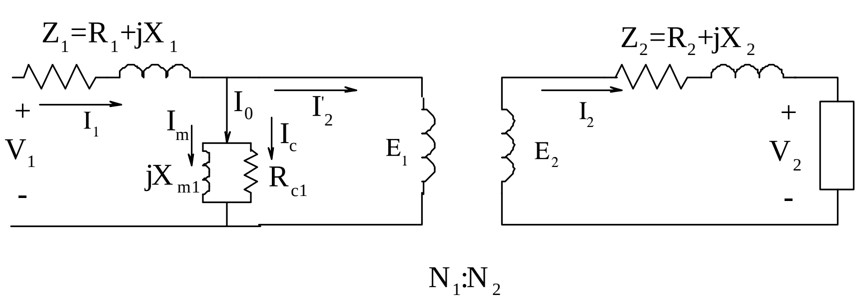
$$\frac{E_1}{E_2} = \frac{I_2}{I_2'} = \frac{N_1}{N_2}$$

I.e. in the ideal transformer, the ratio of currents and voltages is the turns ratio (though the current ratio is inversely so). The ideal transformer is shown below:

In a real transformer, there would be some leakage flux, also some losses. There is also a magnetizing current. These are simulated by adding components to the ideal model, thus a real transformer would be as shown below, this is the T-equivalent model or equivalent circuit.



The ideal tranformer



Equivalent circuit of a power transformer

Assuming the load on an ideal transformer has an impedance Z_2, then we have: $Z_2 = \dfrac{E_2}{I_2}$.

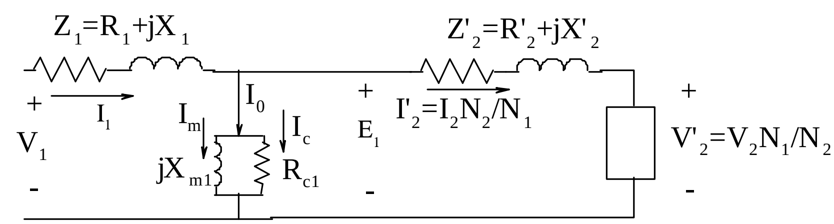
The impedance seen at the left of the ideal transformer is

$Z_1 = \dfrac{E_1}{I_2'} = \dfrac{E_2 \dfrac{N_1}{N_2}}{I_2 \dfrac{N_2}{N_1}} = \left(\dfrac{N_1}{N_2}\right)^2 \dfrac{E_2}{I_2} = \left(\dfrac{N_1}{N_2}\right)^2 Z_2$. Thus impedances are transformed as the turns ratio squared (or the inverse turns ratio squared going the other way). Let $a = \dfrac{N_1}{N_2}$.

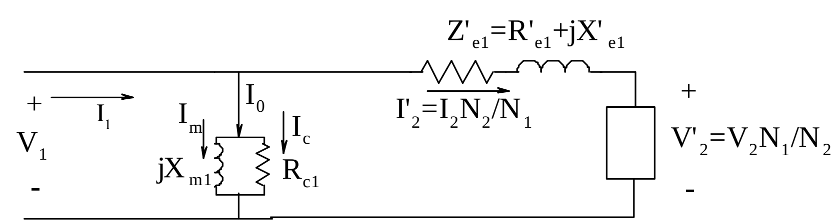
Then: $E_2 = V_2 + Z_2 I_2$. But, $E_1 = a E_2$ and $I_2 = a I_2'$, therefore:

$$\begin{aligned} E_1 &= a V_2 + a^2 Z_2 I_2' \\ &= V_2' + Z_2' I_2' \end{aligned}$$

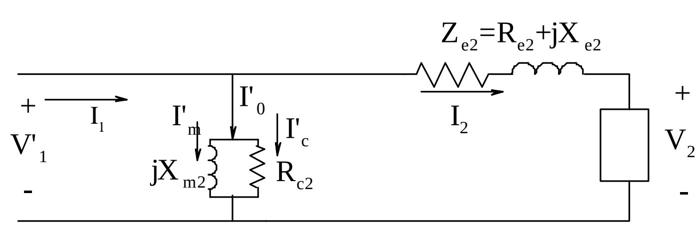
where $Z_2' = R_2' + jX_2' = a^2 R_2 + a^2 X_2$.
Using these transformations, the model of the transformer will be as shown in the figure below:



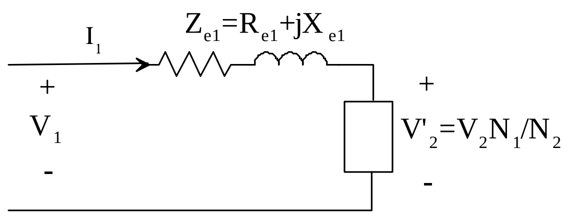
This is then approximated with the figure below:

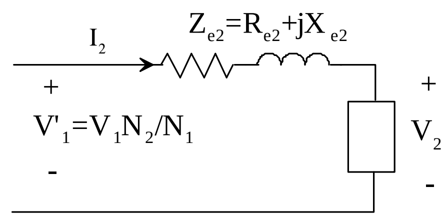


It is easier to perform computations with the latter figure. Note that this approximate equivalent model can be either on the left side (primary) as shown above, or the right side (secondary) as shown below.



Furthermore, if we neglect the shunt parts (which are relatively of high impedance) the result is only an RL equivalent as shown below. In most applications, only the reactance X is kept since R is so much smaller it is neglected.

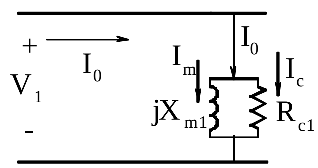




Simplified models referred to one side.

3.7 Determination of Equivalent Circuit Parameters

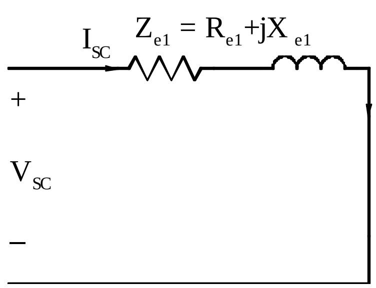
The OC and SC tests are used to determine the equivalent circuit parameters. This is done using the approximate model of figure 3.12 (top of page 68). In the OC test, one side is excited to rated voltage V_1 (usually the low voltage side) and the other side is open circuited. The input power P_0 and current I_0 are measured. Since the current is small compared to rated value, the losses in the series arms of the model may be neglected and it is assumed that the power lost is the "core losses" (the iron core losses) in the shunt components R_{c1} and X_{m1} which are in parallel. The computations required are as follows: $R_{c1} = \frac{V_1^2}{P_0}$,



OC Test

$I_c = \frac{V_1}{R_{c1}}$, $I_m = \sqrt{I_0^2 - I_c^2}$ and $X_{m1} = \frac{V_1}{I_m}$.

In the SC test a reduced voltage V_{SC} is applied to one side while the other side is shorted. The input current I_{SC} and input power P_{SC} are measured. In this test the applied voltage is raised till the current on the short circuit side reaches rated value. Note that only a few percent of rated voltage is needed to produce rated currents under this SC condition.



Short Circuit Test

Since the currents are so high and the voltage applied is so low, the shunt branch may be neglected, and the series branch is where the losses are (approximately). Note that the series branch is the "equivalent" values of the model $Z_{e1} = R_{e1} + jX_{e1}$ (see figure 3.15, top of page 70). The equations needed to compute these parameters are: $Z_{e1} = \frac{V_{SC}}{I_{SC}}$, $R_{e1} = \frac{P_{SC}}{I_{SC}^2}$ and $X_{e1} = \sqrt{Z_{e1}^2 - R_{e1}^2}$.

3.8 Transformer Performance

If we have the approximate equivalent model from the OC and SC tests, we can measure transformer performance. The efficiency of these transformers tends to be high (above 95% efficient). The efficiency h is computed as follows:

$$h = \frac{power\ \ out}{power\ \ in} = \frac{n \times S \times PF}{(n \times S \times PF) + n^2 \times P_{cu} + P_c}$$

where n is the fraction of the full-load power, S is the rated apparent power or nominal power in volt-ampere, and P_{cu} is the full load copper loss. For a three phase transformer these are given by: $S = 3|V_2||I_2|$ and $P_{cu} = 3R_{e2}|I_2|^2$. Also, P_c is the iron loss at rated voltage. As the book says, "it can be easily shown that maximum efficiency occurs when copper losses equal iron losses at n per-unit loading…" see page 71!

At any rate, we do not need these details. There is also a program called "trans" which we have seen before that solves these equations for efficiency. Other important equations are those for percent regulation. These are referred to either side and have to do with voltage drop at full load compared to no load voltage. See equations on bottom of page 71.

Example 3.4

Run Trans with the given data! see page 72.

3.9 Three-Phase Transformer Connections

$Y-Y$: This is rarely used due to third harmonics. Often a third "tertiary" winding is added connected in Δ to provide a path for the third harmonic current thus allowing the voltage to remain sinusoidal and almost distortion free. Voltage out is in phase with voltage in.

$\Delta-\Delta$: This allows for the third harmonic, it lacks the neutral connection. It has the advantage that one bank (assuming three banks are used) of transformers can be removed for repairs, and it will continue to operate (at reduced power) till the repair is completed. It is then connected as a $V-V$ or simply V-connection.

$\Delta-Y$: Is used frequently to raise voltage. Note that there is a connection voltage gain (in addition to the inherent turns ratio gain). There is also a phase shift of thirty degrees. The neutral is often grounded.

$Y-\Delta$: Is used to lower the voltage from high transmission values. Note that it also compensates for phase shifts on a $\Delta-Y$ connection thus bringing the line back in phase.

3.9.1 The Per-Phase Model of a Three-Phase Transformer

Usually the low voltage side is designated as the X side and the high voltage side as the H side. Thus V_{XP} refers to the phase voltage on the low side. In the case of $\Delta - \Delta$ and $Y - Y$ connections the phase does not change going from one side to the other. Also, the voltage gain is essentially the turns ratio. This is not the same for a $\Delta - Y$ or a $Y - \Delta$ connection. Take for example the $\Delta - Y$ connection. First of all there is a 30° phase shift. It is the standard (American Standards Association) to make the connection such that the line voltage on the HV side leads the corresponding line voltage on the LV side by 30° regardless of which side the Y or the Δ is on. Second, the Y-side has a connection gain of $\sqrt{3}$. Assume the Y-side is the HV side, thus we have for a $\Delta - Y$ connection: (let $a = \frac{N_H}{N_X} = \frac{V_{HP}}{V_{XP}}$)

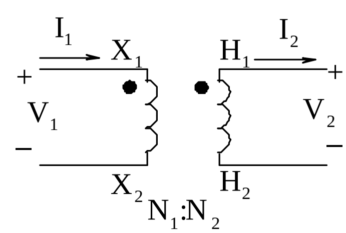
$$V_{HL} = \sqrt{3} V_{HP}$$
$$V_{XL} = V_{XP}$$

thus the ratio of the line-to-line voltages of a $\Delta - Y$ assuming the Y is the high voltage side is $\frac{V_{HL}}{V_{XL}} = \sqrt{3} \frac{V_{HP}}{V_{XP}} = \sqrt{3} a$. I.e. the total gain is the connection gain multiplied by the turns ratio.

When dealing with a $\Delta - Y$ or a $Y - \Delta$ connection, it is best to transform the delta to a Y then use a per-phase diagram. See figure 3.19 page 77.

3.10 Autotransformers

When the two windings of a transformer are connected together it becomes an autotransformer. Note that the two sides are now no longer isolated from each other as in a conventional transformer. This may be undesirable. If the isolation is not an important feature, then this connection has several advantages as we show below. The two winding transformer is shown to the right. Below that we show the two sides connected into an autotransformer. Note that now the two sides are no longer isolated.



The winding voltages and currents are related in the same manner for the autotransformer as for the two-winding transformer, thus: $\frac{V_1}{V_2} = \frac{N_1}{N_2} = a$ and $\frac{I_2}{I_1} = \frac{N_1}{N_2} = a$. From the

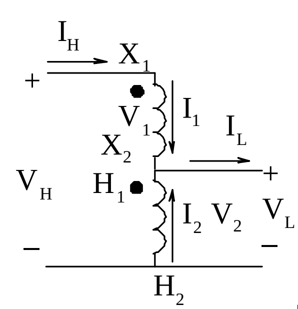


figure we have: $V_H = V_2 + V_1 = V_2 + \frac{N_1}{N_2} V_2 = (1+a) V_L$. Thus we have $\frac{V_H}{V_L} = 1+a$. Since we have the ideal transformer (it is a good approximation in this case) we have $N_2 I_2 = N_1 I_1$ or $N_2 (I_L - I_1) = N_1 I_1$ or $I_L = \frac{N_1 + N_2}{N_2} I_1$, therefore $\frac{I_L}{I_H} = 1+a$. If we now compare the power rating of the two transformers we have:
$\frac{S_{auto}}{S_{2-w}} = \frac{(V_1+V_2)I_1}{V_1 I_1} = 1 + \frac{N_2}{N_1} = 1 + \frac{1}{a}$. If $a < 1$ then the power rating to the autotransformer is higher than the conventional two winding transformer. This is known as the *power rating advantage* of the autotransformer. In addition to this, the autotransformer has the advantages of higher efficiency, lower internal impedance and smaller size (for the same rating). In three phase systems this transformer is used frequently provided the value of the voltage ratio from one side to the other does not exceed three (i.e. $a \leq 3$).

Example 3.5

A two-winding transformer is rated 60 kVA, 240/1200 V, 60 Hz. When operated as a conventional two winding transformer at rated load, 0.8 power factor, its efficiency is 0.96. This transformer is to be used as a 1440/1200 V step-down autotransformer.
(a) Assuming ideal transformer find the transformer kVA rating when used as an autotransformer.
(b) Find the effiecnecy with the kVA loading of part (a) and 0.8 power factor.

The Matlab program follows:

```
S_2w = 60;
V1 = 240; V2 = 1200;
I1= S_2w*10^3/V1; I2 = S_2w*10^3/V2;
IL = I1 + I2;
S_auto = V2*IL*10^-3
Ploss = S_2w*0.8*(1 - 0.96)/0.96
Eff = S_auto*.8/(S_auto*0.8 + Ploss)

S_auto =
   360
Ploss =
    2.0000
Eff =
    0.9931
```

We shall skip section 3.12.

Below is a short program to plot the three phase voltages of amplitude one at 60 Hz.

```
f=60;
w=2*pi*f;
t=0:0.0001:0.02;
```

```
T=1000*t;
a=cos(w*t);
b=cos(w*t+2*pi/3);
c=cos(w*t-2*pi/3);
plot(T,a,T,b,T,c),grid, xlabel('milliseconds'),
ylabel('Amplitude'), title('a  is  blue, b  is  red, c  is  green')
```