

**Midterm Examination I**

**Pages: 4 printed sides**

Name: \_\_\_\_\_  
 Student ID: \_\_\_\_\_

Time allotted: **50 minutes.**  
 Maximum score: **40 points**

*Closed book, closed notes, no calculators.*

**University rules dictate strict penalties for any form of academic dishonesty. Looking sideways will be penalized. Look at only your own exam at all times.**

There are 7 questions, some with subparts. Read them *carefully* to avoid throwing away points!!  
 To be considered for partial credit, you must show intermediate steps.

1. Convert the binary number 11011.0101 to decimal. Show your steps. Show only the first two digits after the decimal point. **(3 points).**

$$(11011)_2 = 2^4 + 2^3 + 2^1 + 2^0 = 16 + 8 + 2 + 1 = 27.$$

$$(0.0101)_2 = 2^{-2} + 2^{-4} = \frac{1}{4} + \frac{1}{16} = \frac{5}{16} = 0.31 \text{ // First two digits only.}$$

$$\text{Answer} = (27.31)_{10}$$

2. Convert the following decimal numbers to binary: (a) 37 (b) 0.9. Show your steps. **(7 points)**

(a)

37		
18		1
9		0
4		1
2		0
1		0
0		1

$$37 = (100101)_2$$

(b)

0.9	* 2	=	0.8	+	1
0.8	* 2	=	0.6	+	1
0.6	* 2	=	0.2	+	1
0.2	* 2	=	0.4	+	0
0.4	* 2	=	0.8	+	0

*// 0.8 (second row) repeats => stop.*

$$0.9 = 0.\overline{11100} \text{ // last four digits repeat.}$$

3. Convert  $(10100101010)_2$  to hexadecimal form. **(2 points)**

Here we group the binary digits into groups of 4 from the right:

<u>101</u>	<u>0010</u>	<u>1010</u>	
5	2	A	= (52A) <sub>16</sub> or 0x52A.

4. Show the steps in performing the subtraction  $20 - 45$  in a 7-bit computer that uses 2's complement representation. Show the binary answer in the computer. State in words the check used for an overflow. Is there an overflow?

**(8 points)**

In a computer with two complement representation, positive numbers are represented as themselves, and negative numbers as the 2's complement of their absolute values. Hence:

$$20 = 10100 = 0010100 \text{ in 7 bits.}$$

$$45 = 101101 = 0101101 \text{ in 7 bits.}$$

$$\Rightarrow -45 = 2\text{'s complement of } 0101101 = 1010010 + 1 \text{ // Flip the bits + 1.}$$

$$= 1010011.$$

$20 - 45 = 20 + (-45)$ . This is done below:

$$\begin{array}{r} 0^1010100 \text{ // } 20 \\ +1\ 010011 \text{ // } -45 \\ \hline 1\ 100111 \\ \text{Answer} = 1100111. \end{array}$$

$$1\ 100111$$

$$1\ 100111$$

$$\text{Answer} = 1100111.$$

Check used for overflow:

There is an overflow if carry into sign bit (here 0)  $\neq$  carry out of sign bit (here 0).

Here they are equal, so there is NO OVERFLOW.

5. Simplify the boolean function  $F(x,y,z) = \Pi M(1,3,6,7)$  using algebraic manipulation.

**(6 points)**

Expanding the function in maxterm canonical form is clumsy, so we first convert to minterm canonical form:

$$F(x,y,z) = \Sigma M(0,2,4,5)$$

$$= x'y'z' + x'yz' + xy'z' + xy'z$$

$$= x'z'(y'+y) + xy'(z'+z)$$

$$= x'z' + xy'$$

6. For inputs x and y, write the function the XNOR operator represents. Derive using Boolean manipulation if XNOR is associative. (7 points)

Let us represent XNOR =

$$x \text{ XNOR } y = (x \text{ XOR } y)'$$

$$= (xy' + x'y)' \text{ //Complement by taking the remaining minterms.}$$

Associative if  $x \text{ XNOR } (y \text{ XNOR } z) = (x \text{ XNOR } y) \text{ XNOR } z$

$$\text{LHS} = x \text{ XNOR } (yz + y'z')$$

$$= x(yz + y'z') + x'(yz' + y'z) \text{ //Complement by taking the remaining minterms.}$$

$$= xyz + xy'z' + x'yz' + x'y'z$$

$$\text{RHS} = (x \text{ XNOR } y) \text{ XNOR } z$$

$$= (xy + x'y') \text{ XNOR } z$$

$$= (xy + x'y')z + (xy' + x'y)z'$$

$$= xyz + x'y'z + xy'z' + x'yz' \text{ //This is the same as LHS after reordering terms.}$$

$$\Rightarrow \text{LHS} = \text{RHS}$$

$\Rightarrow$  XNOR is associative.

7. The equality checking function for single bits, EC-BIT( $x,y$ ) is defined as follows, where  $x,y$  are single bit inputs:

$$\text{EC-BIT}(x,y) = \begin{cases} 1 & \text{if } x \text{ equals } y \\ 0 & \text{otherwise} \end{cases} \quad (2 + 2 + 3 = 7 \text{ points})$$

- (a) Write a Boolean expression for EC-BIT( $x,y$ ) in terms of  $x,y$ .

$$\text{EC-BIT}(x,y) = xy + x'y' \quad // \text{ Either both 1 or both 0.}$$

- (b) Let EC-NUM( $X,Y$ ) be a function that takes as input two 5-bit binary numbers  $X$  and  $Y$ , where  $X = x_4, x_3, x_2, x_1, x_0$ , and  $Y = y_4, y_3, y_2, y_1, y_0$ . It returns 1 if  $X=Y$ . How many rows does the truth table for EC-NUM have?

$$\begin{aligned} \text{Number of bit inputs} &= 5 + 5 = 10. \\ \Rightarrow \text{Number of rows} &= 2^{10} = 1024 \text{ rows.} \end{aligned}$$

- (c) Write a boolean expression for EC-NUM( $X,Y$ ) in terms of EC-BIT( $x_i,y_i$ ) for ( $0 \leq i \leq 4$ ).  
(Hint: Do not use any formal method; simply reason about this intuitively.)

$$\begin{aligned} \text{EC-NUM}(X,Y) &= \\ &\text{EC-BIT}(x_4,y_4) \cdot \text{EC-BIT}(x_3,y_3) \cdot \text{EC-BIT}(x_2,y_2) \cdot \text{EC-BIT}(x_1,y_1) \cdot \text{EC-BIT}(x_0,y_0) \end{aligned}$$

$$\text{Above can also be written as } \prod_{i=0}^{i=4} \text{EC-BIT}(x_i,y_i)$$