ENEE 244 (01**). Spring 2006

Homework 3

Due back in class on Monday, March 27.

1. Does the function xy'z imply the function xy' + yz'?

A function f implies g if when f is true, then g is true.
Here, when xy'z is true, then x=1, y=0, z=1.
For these values the xy' + yz' function is always true since the first term is always true.
⇒ Yes, it does imply.

2. Does the function x imply the function xy' + xz?

To show that it does not imply, it is enough to find an assignment of values to x,y,and z that makes the first function true but not the second.
Here, suppose x=1, y=1, z=0. For this input, x is true, but xy' + xz is not.
⇒ No, it does not imply.

//Note about questions on implication: You can always find if a function implies another by drawing the truth table for both and seeing if the set of 1's for the second function is a superset of the first. However you can sometimes come up with a faster answer by using an intuitive proof like the two answers above show. There is no set way of deriving these intuitive proofs. You can come up with many intuitive proofs based on intelligent observation.

3. Does (a) xy' subsume xy'z'; and (b) x + y' subsume x?

Term T1 subsumes T2 iff all the literals of T2 are also in T1, for both product and sum terms. So using this rule:

(a) No.

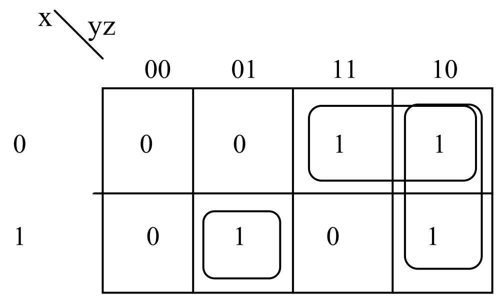
(b) Yes.

4. Minimize the following functions using K-maps: (a) F(x,y,z) = xy'z + x'yz + xyz' + x'yz'; (b) F(w,x,y,z) = Σm(1,3,4,5,8,9,10,13).

(a) F(x,y,z) = xy'z + x'yz + xyz' + x'yz'.

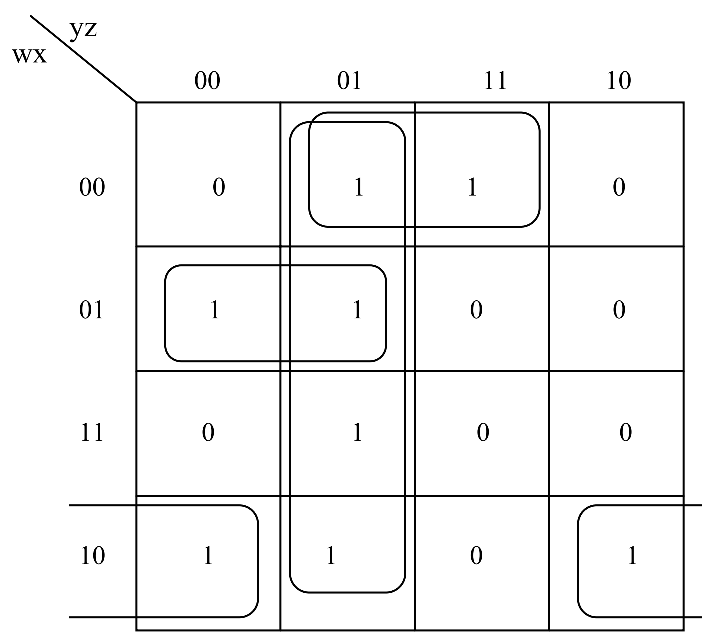
Since this function is in canonical form, we can directly fill the K-map instead of going through the truth table.

K-map:



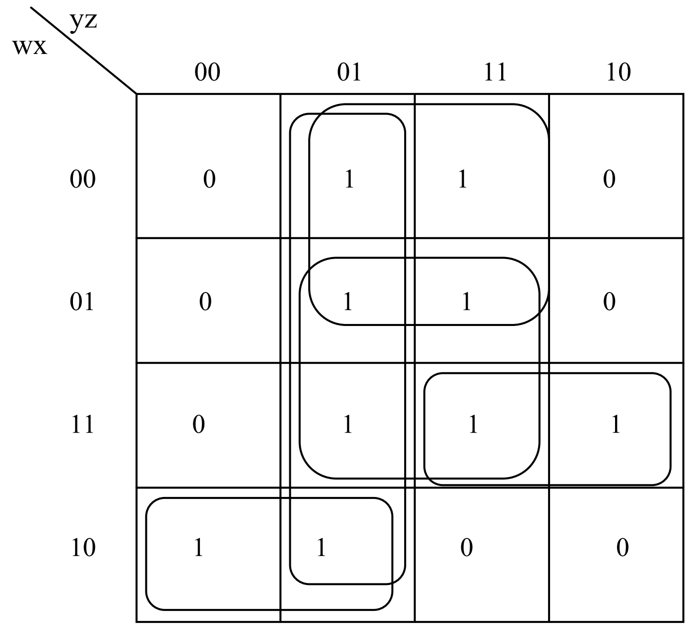
F_{min} = x’y + xy’z + yz’

(b) F(w,x,y,z) = Σm(1,3,4,5,8,9,10,13).



Here all the Prime implicants shown are essential.
(There is one prime implicant covering m8,m9 which is not essential, but it is not needed for covering all minterms, and hence is not shown.)
F_{min} = w’x’z + y’z + w’xy’ + wx’z’

5. Consider the function F(A,B,C,D) = Σm(1,3,5,7,8,9,13,14,15). Draw all its prime implicants on a K-map. Which ones are non-essential? How many possible minimum sum-of-products forms are there and what are they?

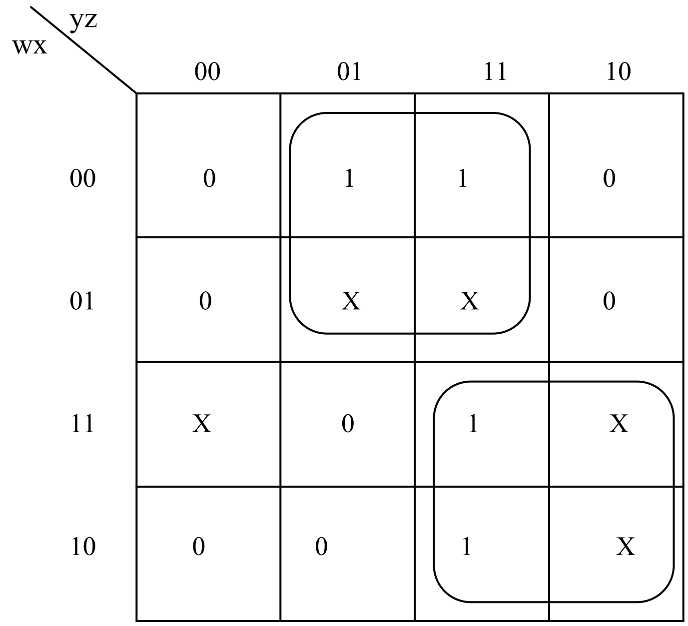


Prime implicants:
m1,m3, m5,m7 : Essential (induced by m3) *// term is w'z'*
m1,m5,m13,m9: Non-essential (all minterms included in other PIs). *// term is y'z*
m5,m7,m13,m15: Non-essential (all minterms included in other PIs). *// term is xz*
m14,m15: Essential (induced by m14) *// term is wxy*
m8,m9: Essential (induced by m8) *// term is wx'y'*

Minimum functions contain all essential PIs and only those non-essential PIs needed to cover minterms not covered by essentials.

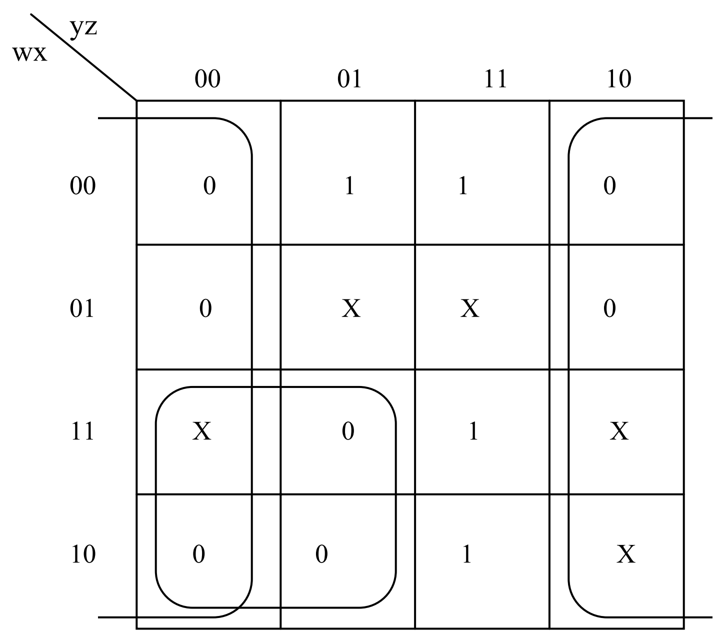
Minterms not covered by essential PIs: only m13.
To cover m13, either one of the two non-essential PIs is enough. Thus, there are two minimum forms, which are:
(i) w'z' + wxy + wx'y' + y'z
(ii) w'z' + wxy + wx'y' + xz
Both the above forms are equivalent and either can be used to implement the function.

6. Minimize the function $F(w,x,y,z) = \Sigma m(1,3,11,15) + dc(5,7,10,12,14)$.



The minimum form is obtained by setting m5,m7,m10, m14 to 1, and setting m12 to 0.
$F_{min} = w'z + wy$.

7. Repeat question 6, but find the minimum in terms of product-of-sums. Is your answer functionally equivalent to the minimum function you obtained in question 6?



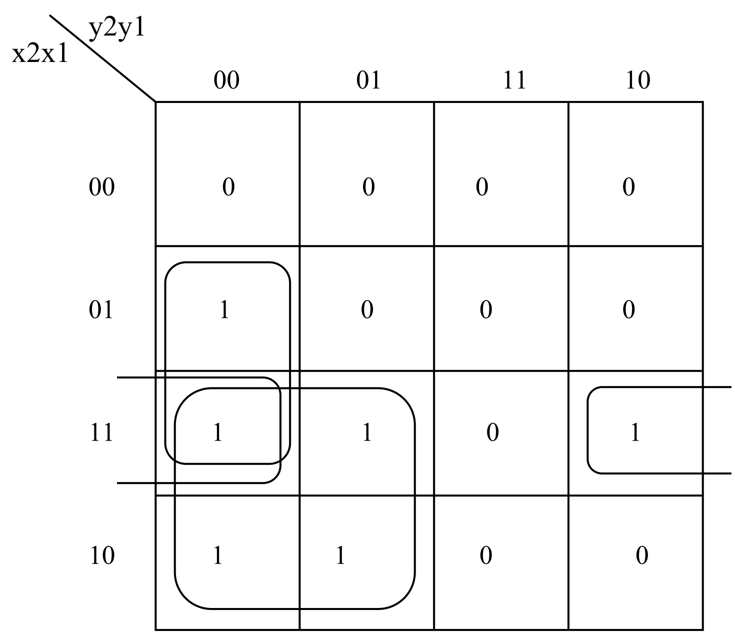
To find the minimum product-of-sums, we combine the zeros instead. In the above, m10,m12, and m14 are set to 0, and m5, m7 are set to 1.
$F_{min}' = z' + wy'$.
$\Rightarrow F_{min} = (z' + wy')' = (z')'.(wy')' = z(w' + y)$.
It is interesting to note that since the don't cares are assigned differently, this is NOT THE SAME FUNCTION as the result of Q.6.

8. Design a circuit that compares two binary numbers X and Y, each having two bits (X=x2x1; Y=y2y1). It has a single output c which is supposed to be 1 when X > Y, and 0 otherwise. Do not use adders/subtractors for this design; instead use a fundamental design process. Draw the minimum sum-of-products circuit using the correct circuit symbols for all gates. Assume complemented inputs are NOT available, but multiple input gates are. What is the delay of this circuit, assuming the delay for each gate is *d*?

Let us first make the truth table from the English-language specification above.

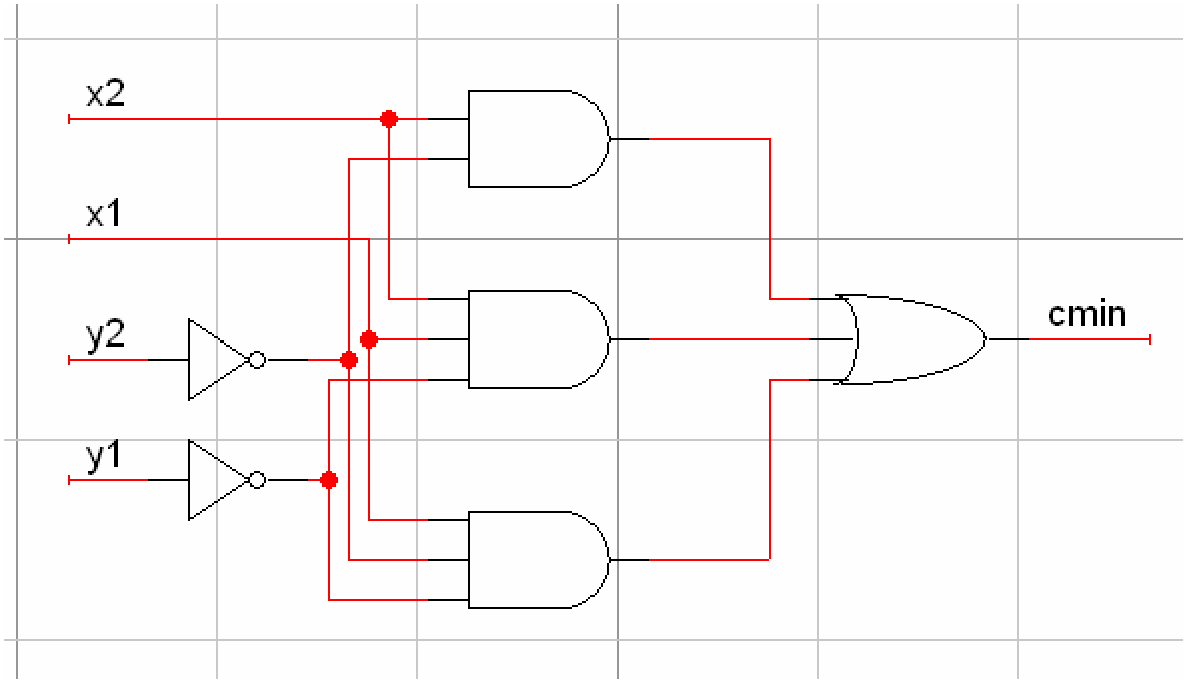
x2	x1	y2	y1	c
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	0
1	0	1	1	0
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

Thus, c=Σm(4,8,9,12,13,14).



Thus, c_{min} = x2y2’ + x2x1y1’ + x1y2’y1’.

<u>Circuit</u>:



The delay of this circuit is *3d* since there are three levels of gates.