Relaxation time

Suppose you were to place a quantity of electric charge Q in doped Si. The charge is initially contained in a small volume v_0 of radius a. The Si has dielectric constant ϵ and conductivity σ . Since like charges repel the charge would spread itself out as much as possible and, after some time, most of it would have left the initial volume. How long would it take for the amount of charge in v_0 to be reduced by 1/e? This time can be found by using the equation of continuity $\frac{d\rho}{dt} + \nabla \cdot J = 0$ and $J = \sigma E$. Since $\nabla \cdot E = \frac{\rho}{\epsilon}$ we immediately have $\frac{d\rho}{dt} + \frac{\sigma}{\epsilon}\rho = 0$

which, when integrated, gives us $\rho = \rho_0 \exp^{-\frac{\sigma}{\epsilon}t} = \rho_0 \exp^{-\frac{t}{\tau_e}}$ where $\tau_{\epsilon} = \frac{\epsilon}{\sigma}$ is the dielectric

time constant. For n-type Si with doping $N_d = 10^{17}$, $\boldsymbol{\sigma} = qn_o \boldsymbol{\mu}_e = 24 (\boldsymbol{\Omega} - cm)^{-1}$ while the dielectric constant of Si is 11.8 $\boldsymbol{\epsilon}_o \approx 10^{-11} \text{ F cm}^{-1}$, so $\frac{\boldsymbol{\epsilon}}{\boldsymbol{\sigma}} \approx 4 \times 10^{-13} \text{ sec}^{-1}$. Thus the relaxation

time is very short.