Equation of continuity

The easiest way to understand the equation of continuity is to consider a gas, which consists of a very large number of particles cm^{-3}, so that we can define a density. Now, take some small volume of gas in a volume dV = Adx, where A is the area of a side of the volume perpendicular to the x-axis. The mass of material in the volume is the $\rho\, dV = \rho\, Adx$. Let the flux of gas molecules into the volume at x be J(x) and the flow out of the volume at x + dx be J(x + dx)) (J has units of gm $cm^{-2}\, sec^{-1}$). Then the time rate of change of the mass due to the net flow out of the volume will be [J(x) - J(x + dx)] A = $-\frac{\partial J(x)}{\partial x}\, dx\ A$. The rate of change of the mass of gas in the volume due to the change in density is $\frac{\partial \rho}{\partial t}\, dV = \frac{\partial \rho}{\partial t}\, dx\ A$. The total rate of change of the mass of gas must be zero since matter is conserved, so

$$\frac{\partial \rho}{\partial t}\, dx\ A = -\frac{\partial J}{\partial x}\, dx\ A \quad \text{or} \quad \frac{\partial \rho}{\partial t} + \frac{\partial J}{\partial x} = 0.$$