Density distribution

An example: finding the density distribution of holes in n-type Si when there is a source of electron-hole pairs given by $g(x)$ = constant = G over part of the length of the Si. Assume the Si is doped with donors at a density N_d and that the Si has a length W. Assume also that $g(x) = 0$ for $x \leq W1$ and for $x \geq W2$ and that there are metallic contacts at x = 0 and x = W so that $p(0) = p(W) = 0$.

For $x \leq W1$ and for $x \geq W2$ the equation for the holes is $\frac{d^2p}{dx^2} - \frac{p}{L_h^2} = 0$, with boundary conditions $p(0) = p(W) = 0$. For $W1 < x < W2$ we have $\frac{d^2p}{dx^2} - \frac{p}{L_h^2} = -\frac{G}{D_h}$. A condition we can impose on the carrier densities is that they and their first derivatives be continuous functions of position, because if they were not that would imply discontinuities in the current density, which would be unphysical.

In the regions where $g = 0$ the solutions are $A\ e^{x/L_h} + B\ e^{-x/L_h}$ for $x \leq W1$ and $C\ e^{x/L_h} + D\ e^{-x/L_h}$ for $x \geq W2$. At x = 0 we have B = -A so that $p(x) = 2A \sinh(x/L_h)$. At x = W we have $D = -C\ \exp^{-2W/L_h}$ and so $p(x) = C\ \exp^{W/L_h} \sinh\left(\frac{x - W}{L_h}\right)$.

Where $g(x) = G$ we have a non-homogeneous differential equation. The homogeneous part is the same as before and so the solution is again $E \sinh(x/L_h)$. To these we add a particular solution equal to a constant, so $p(x) = E \sinh(x/L_h) + \text{const}$. When this is put into the differential equation we get $\frac{E}{L_h^2} \sinh\left(\frac{x}{L_h}\right) - \frac{1}{L_h^2}\left(\sinh\left(\frac{x}{L_h}\right) + \text{const}\right) = -\frac{G}{D_h}$

so the value of the constant is $\text{const} = \frac{GL_h^2}{D_h}$. We then require that p(x) and $\frac{dp}{dx}$ be continuous at x = W1 and at x = W2, which gives the values for A, C and E in terms of W1 and W2. Once these constants are known the current densities can be found for all values of x, as well as the electric field in the Si.